

Tutorium Mathe 1 MT

Übungsblatt 2:

Lösungen

1. Schreiben Sie die folgenden komplexen Zahlen jeweils in allen drei Schreibweisen

$$\text{a) } 2+i = \sqrt{5} \cdot (\cos(27^\circ) + i \sin(27^\circ)) = \sqrt{5} \cdot e^{i27^\circ}$$

$$\text{b) } 4-i8 = \sqrt{80} \cdot (\cos(297^\circ) + i \sin(297^\circ)) = \sqrt{80} \cdot e^{i297^\circ}$$

$$\text{c) } -6i = 6 \cdot (\cos(270^\circ) + i \sin(270^\circ)) = 6 \cdot e^{i270^\circ}$$

$$\text{d) } 4+i4 = \sqrt{32} \cdot (\cos(45^\circ) + i \sin(45^\circ)) = \sqrt{32} \cdot e^{i45^\circ}$$

$$\text{e) } i = \cos(90^\circ) + i \sin(90^\circ) = e^{i90^\circ}$$

$$\text{f) } (4+i)^2 = 15+i8 = 17 \cdot (\cos(28^\circ) + i \sin(28^\circ)) = 17 \cdot e^{i28^\circ}$$

2. Berechnen Sie den Betrag der folgenden komplexen Zahlen

$$\text{a) } |4+i5| = \sqrt{4^2+5^2} = \sqrt{41} \approx 6,4$$

$$\text{b) } \left| \frac{1}{2} + i4 \right| = \sqrt{\frac{1^2}{2} + 4^2} = \sqrt{\frac{65}{4}} \approx 4,03$$

$$\text{c) } |(1+i) \cdot (2+i5)| = |-3+i7| = \sqrt{58} \approx 7,62$$

$$\text{d) } \left| \frac{1}{3+i7} \right| = \left| \frac{3-i7}{58} \right| = \left| \frac{3}{58} - i \frac{7}{58} \right| = \sqrt{\frac{1}{58}} \approx 0,13$$

3. Berechnen Sie Betrag und Argument.

$$\text{a) } |(4+i5) - (1+i4)| = |3+i| = \sqrt{10} \quad \arg(3+i) = 18^\circ$$

$$\text{b) } |3 \cdot e^{i\pi}| = 3 \quad \arg(3 \cdot e^{i\pi}) = \pi = 180^\circ$$

$$\text{c) } |-64i| = 64 \quad \arg(-64i) = 270^\circ$$

$$\text{d) } \left| 7 \cdot \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right) \right| = 7 \quad \arg\left(7 \cdot \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right) \right) = \frac{\pi}{2} = 90^\circ$$

4. Berechnen Sie die folgenden Ausdrücke.

$$\text{a) } (2+i4) - (3+i2) = -1+i2$$

$$\text{b) } (4+i) + (3+i7) = 7+i8$$

$$\text{c) } (3+i2) \cdot (1+i2) = -1+i8$$

$$\text{d) } 4 \cdot (3+i) - (4+i2) = (12+i4) - (4+i2) = 8+i2$$

$$\text{e) } ((2+i4) + (4+i)) \cdot (3 \cdot (2+i) - (1+i)) = (6+i5) \cdot (5+i2) = 20+i37$$

$$\text{f) } \frac{1}{1+i5} = \frac{1}{26} - i \frac{5}{26}$$

$$g) \frac{(4+i2)+(2+i4)}{1+i5} = \frac{6+i6}{1+i5} = \frac{(6+i6) \cdot (1-i5)}{26} = \frac{36-i24}{26} = \frac{18}{13} - i \frac{12}{13}$$

$$h) \frac{(2+i7)^2}{(3+i2) \cdot (3-i2)} = \frac{-45+i28}{13} = -\frac{45}{13} + i \frac{28}{13}$$

$$i) e^{i\pi} + 1 = \cos(\pi) - i \sin(\pi) + 1 = -1 + 1 = 0$$

5. Berechnen Sie alle Nullstellen der Funktionen.

$$a) x^2 + 4x - 6 \Rightarrow x_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 + 6} \Rightarrow x_1 = -2 - \sqrt{10}, x_2 = -2 + \sqrt{10}$$

$$b) x^3 + 2x^2 + 5x = x \cdot (x^2 + 2x + 5) \Rightarrow x_1 = 0$$

$$x^2 + 2x + 5 = 0 \Rightarrow x_{2,3} = -1 \pm \sqrt{1-5} \Rightarrow x_2 = -1 - i2, x_3 = -1 + i2$$

$$x^4 + 2x^2 + 40 = z^2 + 2z + 40 \quad \text{mit } z = x^2 \Rightarrow x = \pm \sqrt{z}$$

$$c) z_{1,2} = -1 \pm \sqrt{1-40} \Rightarrow z_1 = -1 - i \cdot \sqrt{39}, z_2 = -1 + i \cdot \sqrt{39}$$

$$\Rightarrow x_1 = -\sqrt{-1 - i \cdot \sqrt{39}}, x_2 = \sqrt{-1 - i \cdot \sqrt{39}}, x_3 = -\sqrt{-1 + i \cdot \sqrt{39}}, x_4 = \sqrt{-1 + i \cdot \sqrt{39}}$$

6. Berechnen Sie die Lösung der komplexen Gleichung. Geben Sie dazu jeweils zunächst den Betrag und die Argumente der Lösung an.

a)

$$x^4 = 256i$$

$$|x| = \sqrt[4]{|256i|} = \sqrt[4]{256} = 4$$

$$\arg(x_1) = \frac{\arg(256i)}{4} + \frac{0 \cdot 2\pi}{4} = \frac{\frac{\pi}{2}}{4} = \frac{\pi}{8} \triangleq 22,5^\circ$$

$$\arg(x_2) = \frac{\arg(256i)}{4} + \frac{2\pi}{4} = \frac{\pi}{8} + \frac{4\pi}{8} = \frac{5\pi}{8} \triangleq 112,5^\circ$$

$$\arg(x_3) = \frac{\arg(256i)}{4} + \frac{4\pi}{4} = \frac{\pi}{8} + \frac{8\pi}{8} = \frac{9\pi}{8} \triangleq 202,5^\circ$$

$$\arg(x_4) = \frac{\arg(256i)}{4} + \frac{6\pi}{4} = \frac{\pi}{8} + \frac{13\pi}{8} = \frac{14\pi}{8} \triangleq 292,5^\circ$$

Lösungen:

$$x_1 = 4 \cdot e^{i22,5^\circ}$$

$$x_2 = 4 \cdot e^{i112,5^\circ}$$

$$x_3 = 4 \cdot e^{i202,5^\circ}$$

$$x_4 = 4 \cdot e^{i292,5^\circ}$$

b)

$$x^6 = 729i$$

$$|x| = \sqrt[6]{|729i|} = \sqrt[6]{729} = 3$$

$$\arg(x_1) = \frac{\arg(729i)}{6} + \frac{0 \cdot 2\pi}{6} = \frac{\pi}{6} = \frac{\pi}{12} \triangleq 15^\circ$$

$$\arg(x_2) = \frac{\arg(729i)}{6} + \frac{2\pi}{6} = \frac{\pi}{6} + \frac{4\pi}{12} = \frac{5\pi}{12} \triangleq 75^\circ$$

$$\arg(x_3) = \frac{\arg(729i)}{6} + \frac{4\pi}{6} = \frac{\pi}{6} + \frac{8\pi}{12} = \frac{9\pi}{12} \triangleq 135^\circ$$

$$\arg(x_4) = \frac{\arg(729i)}{6} + \frac{6\pi}{6} = \frac{\pi}{6} + \frac{12\pi}{12} = \frac{14\pi}{12} \triangleq 195^\circ$$

$$\arg(x_5) = \frac{\arg(729i)}{6} + \frac{8\pi}{6} = \frac{\pi}{6} + \frac{16\pi}{12} = \frac{17\pi}{12} \triangleq 265^\circ$$

$$\arg(x_6) = \frac{\arg(729i)}{6} + \frac{10\pi}{6} = \frac{\pi}{6} + \frac{20\pi}{12} = \frac{21\pi}{12} \triangleq 325^\circ$$

Lösungen :

$$x_1 = 6 \cdot e^{i15^\circ}$$

$$x_2 = 6 \cdot e^{i75^\circ}$$

$$x_3 = 6 \cdot e^{i135^\circ}$$

$$x_4 = 6 \cdot e^{i195^\circ}$$

$$x_5 = 6 \cdot e^{i265^\circ}$$

$$x_6 = 6 \cdot e^{i325^\circ}$$

c)

$$x^3 = 5 + i\sqrt{2}$$

$$|x| = \sqrt[3]{25+2} = \sqrt[3]{27} = 3$$

$$\arg(x_1) = \frac{\arg(5+i\sqrt{2})}{3} + \frac{0 \cdot 2\pi}{3} = 16^\circ$$

$$\arg(x_2) = \frac{\arg(5+i\sqrt{2})}{3} + \frac{2\pi}{3} = 16^\circ + 120^\circ = 136^\circ$$

$$\arg(x_3) = \frac{\arg(5+i\sqrt{2})}{3} + \frac{4\pi}{3} = 16^\circ + 240^\circ = 256^\circ$$

Lösungen :

$$x_1 = 3 \cdot e^{i16^\circ}$$

$$x_2 = 3 \cdot e^{i136^\circ}$$

$$x_3 = 3 \cdot e^{i256^\circ}$$

d)

$$x^5 = 5 \cdot e^{i75^\circ}$$

$$|x| = \sqrt[5]{5} \approx 1,38$$

$$\arg(x_1) = \frac{75^\circ}{5} + \frac{0 \cdot 360^\circ}{5} = 15^\circ$$

$$\arg(x_2) = \frac{75^\circ}{5} + \frac{360^\circ}{5} = 87^\circ$$

$$\arg(x_3) = \frac{75^\circ}{5} + \frac{2 \cdot 360^\circ}{5} = 159^\circ$$

$$\arg(x_4) = \frac{75^\circ}{5} + \frac{3 \cdot 360^\circ}{5} = 231^\circ$$

$$\arg(x_5) = \frac{75^\circ}{5} + \frac{4 \cdot 360^\circ}{5} = 303^\circ$$

Lösungen :

$$x_1 = 1,38 \cdot e^{i15^\circ}$$

$$x_2 = 1,38 \cdot e^{i87^\circ}$$

$$x_3 = 1,38 \cdot e^{i159^\circ}$$

$$x_4 = 1,38 \cdot e^{i231^\circ}$$

$$x_5 = 1,38 \cdot e^{i303^\circ}$$

e)

$$x^3 = 27 \cdot e^{i\frac{\pi}{2}}$$

$$|x| = \sqrt[3]{27} = 3$$

$$\arg(x_1) = \frac{\pi}{3} = \frac{\pi}{6} \hat{=} 30^\circ$$

$$\arg(x_2) = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6} \hat{=} 150^\circ$$

$$\arg(x_3) = \frac{\pi}{6} + \frac{4\pi}{3} = \frac{9\pi}{6} \hat{=} 270^\circ$$

Lösungen :

$$x_1 = 3 \cdot e^{i30^\circ}$$

$$x_2 = 3 \cdot e^{i150^\circ}$$

$$x_3 = 3 \cdot e^{i270^\circ}$$