Tutorium Mathe 1 MT

Übungsblatt 2: **Lösungen**

1. Schreiben Sie die folgenden komplexen Zahlen jeweils in allen drei Schreibweisen
   a) $2+i=\sqrt{5}\cdot\left(\cos\left(27°\right)+i\sin\left(27°\right)\right)=\sqrt{5}\cdot e^{i27°}$
   b) $4-i8=\sqrt{80}\cdot\left(\cos\left(297°\right)+i\sin\left(297°\right)\right)=\sqrt{80}\cdot e^{i297°}$
   c) $-6i=6\cdot\left(\cos\left(270°\right)+i\sin\left(270°\right)\right)=6\cdot e^{i270°}$
   d) $4+i4=\sqrt{32}\cdot\left(\cos\left(45°\right)+i\sin\left(45°\right)\right)=\sqrt{32}\cdot e^{i45°}$
   e) $i=\cos\left(90°\right)+i\sin\left(90°\right)=e^{i90°}$
   f) $\left(4+i\right)^2=15+i8=17\cdot\left(\cos\left(28°\right)+i\sin\left(28°\right)\right)=17\cdot e^{i28°}$

2. Berechnen Sie den Betrag der folgenden komplexen Zahlen
   a) $\left|4+i5\right|=\sqrt{4^2+5^2}=\sqrt{41}\approx 6,4$
   b) $\left|\frac{1}{2}+i4\right|=\sqrt{\frac{1}{2}^2+4^2}=\sqrt{\frac{65}{4}}\approx 4,03$
   c) $\left|\left(1+i\right)\cdot\left(2+i5\right)\right|=\left|-3+i7\right|=\sqrt{58}\approx 7,62$
   d) $\left|\frac{1}{3+i7}\right|=\left|\frac{3-i7}{58}\right|=\left|\frac{3}{58}-i\frac{7}{58}\right|=\sqrt{\frac{1}{58}}\approx 0,13$

3. Berechnen Sie Betrag und Argument.

   a) $\left|\left(4+i5\right)-\left(1+i4\right)\right|=\left|3+i\right|=\sqrt{10}$ $\quad\arg\left(3+i\right)=18°$
   b) $\left|3\cdot e^{i\pi}\right|=3$ $\quad\arg\left(3\cdot e^{i\pi}\right)=\pi=180°$
   c) $\left|-64i\right|=64$ $\quad\arg\left(-64i\right)=270°$
   d) $\left|7\cdot\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)\right|=7$ $\quad\arg\left(7\cdot\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)\right)=\frac{\pi}{2}=90°$

4. Berechnen Sie die folgenden Ausdrücke.
   a) $\left(2+i4\right)-\left(3+i2\right)=-1+i2$
   b) $\left(4+i\right)+\left(3+i7\right)=7+i8$
   c) $\left(3+i2\right)\cdot\left(1+i2\right)=-1+i8$
   d) $4\cdot\left(3+i\right)-\left(4+i2\right)=\left(12+i4\right)-\left(4+i2\right)=8+i2$
   e) $\left(\left(2+i4\right)+\left(4+i\right)\right)\cdot\left(3\cdot\left(2+i\right)-\left(1+i\right)\right)=\left(6+i5\right)\cdot\left(5+i2\right)=20+i37$
   f) $\frac{1}{1+i5}=\frac{1}{26}-i\frac{5}{26}$

g) $\frac{(4+i2)+(2+i4)}{1+i5}=\frac{6+i6}{1+i5}=\frac{(6+i6)\cdot(1-i5)}{26}=\frac{36-i24}{26}=\frac{18}{13}-i\frac{12}{13}$

h) $\frac{(2+i7)^2}{(3+i2)\cdot(3-i2)}=\frac{-45+i28}{13}=-\frac{45}{13}+i\frac{28}{13}$

i) $e^{i\pi}+1=\cos(\pi)-i\sin(\pi)+1=-1+1=0$

5. Berechnen Sie alle Nullstellen der Funktionen.

a) $x^2+4x-6 \Rightarrow x_{1,2}-\frac{4}{2}\pm\sqrt{\left(\frac{4}{2}\right)^2+6} \Rightarrow x_1=-2-\sqrt{10}, x_2=-2+\sqrt{10}$

b)
$$x^3+2x^2+5x=x\cdot(x^2+2x+5) \Rightarrow x_1=0$$
$$x^2+2x+5=0 \Rightarrow x_{2,3}=-1\pm\sqrt{1-5} \Rightarrow x_2=-1-i2, x_3=-1+i2$$

c)
$$x^4+2x^2+40=z^2+2z+40 \qquad mit\ z=x^2 \Rightarrow x=\pm\sqrt{z}$$
$$z_{1,2}=-1\pm\sqrt{1-40} \Rightarrow z_1=-1-i\cdot\sqrt{39}, z_2=-1+i\cdot\sqrt{39}$$
$$\Rightarrow x_1=-\sqrt{-1-i\cdot\sqrt{39}}, x_2=\sqrt{-1-i\cdot\sqrt{39}}, x_3=-\sqrt{-1+i\cdot\sqrt{39}}, x_4=\sqrt{-1+i\cdot\sqrt{39}}$$

6. Berechnen Sie die Lösung der komplexen Gleichung. Geben Sie dazu jeweils zunächst den Betrag und die Argumente der Lösung an.

a)
$$x^4=256i$$
$$|x|=\sqrt[4]{|256i|}=\sqrt[4]{256}=4$$
$$\arg(x_1)=\frac{\arg(256i)}{4}+\frac{0\cdot 2\pi}{4}=\frac{\frac{\pi}{2}}{4}=\frac{\pi}{8}\triangleq 22{,}5°$$
$$\arg(x_2)=\frac{\arg(256i)}{4}+\frac{2\pi}{4}=\frac{\pi}{8}+\frac{4\pi}{8}=\frac{5\pi}{8}\triangleq 112{,}5°$$
$$\arg(x_3)=\frac{\arg(256i)}{4}+\frac{4\pi}{4}=\frac{\pi}{8}+\frac{8\pi}{8}=\frac{9\pi}{8}\triangleq 202{,}5°$$
$$\arg(x_4)=\frac{\arg(256i)}{4}+\frac{6\pi}{4}=\frac{\pi}{8}+\frac{13\pi}{8}=\frac{14\pi}{8}\triangleq 292{,}5°$$

Lösungen:

$$x_1=4\cdot e^{i22{,}5°}$$
$$x_2=4\cdot e^{i112{,}5°}$$
$$x_3=4\cdot e^{i202{,}5°}$$
$$x_4=4\cdot e^{i292{,}5°}$$

b)

$$x^6 = 729i$$

$$|x| = \sqrt[6]{|729i|} = \sqrt[6]{729} = 3$$

$$\arg(x_1) = \frac{\arg(729i)}{6} + \frac{0\cdot 2\pi}{6} = \frac{\frac{\pi}{2}}{6} = \frac{\pi}{12} \mathrel{\hat{=}} 15°$$

$$\arg(x_2) = \frac{\arg(729i)}{6} + \frac{2\pi}{6} = \frac{\pi}{12} + \frac{4\pi}{12} = \frac{5\pi}{12} \mathrel{\hat{=}} 75°$$

$$\arg(x_3) = \frac{\arg(729i)}{6} + \frac{4\pi}{6} = \frac{\pi}{12} + \frac{8\pi}{12} = \frac{9\pi}{12} \mathrel{\hat{=}} 135°$$

$$\arg(x_4) = \frac{\arg(729i)}{6} + \frac{6\pi}{6} = \frac{\pi}{12} + \frac{12\pi}{12} = \frac{14\pi}{12} \mathrel{\hat{=}} 195°$$

$$\arg(x_5) = \frac{\arg(729i)}{6} + \frac{8\pi}{6} = \frac{\pi}{12} + \frac{16\pi}{12} = \frac{17\pi}{12} \mathrel{\hat{=}} 265°$$

$$\arg(x_6) = \frac{\arg(729i)}{6} + \frac{10\pi}{6} = \frac{\pi}{12} + \frac{20\pi}{12} = \frac{21\pi}{12} \mathrel{\hat{=}} 325°$$

$Lösungen:$

$$x_1 = 6\cdot e^{i15°}$$

$$x_2 = 6\cdot e^{i75°}$$

$$x_3 = 6\cdot e^{i135°}$$

$$x_4 = 6\cdot e^{i195°}$$

$$x_5 = 6\cdot e^{i265°}$$

$$x_6 = 6\cdot e^{i325°}$$

c)

$$x^3 = 5 + i\sqrt{2}$$

$$|x| = \sqrt[3]{25+2} = \sqrt[3]{27} = 3$$

$$\arg(x_1) = \frac{\arg\left(5+i\sqrt{2}\right)}{3} + \frac{0\cdot 2\pi}{3} = 16°$$

$$\arg(x_2) = \frac{\arg\left(5+i\sqrt{2}\right)}{3} + \frac{2\pi}{3} = 16° + 120° = 136°$$

$$\arg(x_3) = \frac{\arg\left(5+i\sqrt{2}\right)}{3} + \frac{4\pi}{3} = 16° + 240° = 256°$$

$Lösungen:$

$$x_1 = 3\cdot e^{i16°}$$

$$x_2 = 3\cdot e^{i136°}$$

$$x_3 = 3\cdot e^{i256°}$$

d)

$$x^5 = 5 \cdot e^{i75°}$$

$$|x| = \sqrt[5]{5} \approx 1,38$$

$$\arg(x_1) = \frac{75°}{5} + \frac{0 \cdot 360°}{5} = 15°$$

$$\arg(x_2) = \frac{75°}{5} + \frac{360°}{5} = 87°$$

$$\arg(x_3) = \frac{75°}{5} + \frac{2 \cdot 360°}{5} = 159°$$

$$\arg(x_4) = \frac{75°}{5} + \frac{3 \cdot 360°}{5} = 231°$$

$$\arg(x_5) = \frac{75°}{5} + \frac{4 \cdot 360°}{5} = 303°$$

Lösungen:

$$x_1 = 1,38 \cdot e^{i15°}$$

$$x_2 = 1,38 \cdot e^{i87°}$$

$$x_3 = 1,38 \cdot e^{i159°}$$

$$x_4 = 1,38 \cdot e^{i231°}$$

$$x_5 = 1,38 \cdot e^{i303°}$$

e)

$$x^3 = 27 \cdot e^{i\frac{\pi}{2}}$$

$$|x| = \sqrt[3]{27} = 3$$

$$\arg(x_1) = \frac{\frac{\pi}{2}}{3} = \frac{\pi}{6} \mathrel{\hat{=}} 30°$$

$$\arg(x_2) = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6} \mathrel{\hat{=}} 150°$$

$$\arg(x_3) = \frac{\pi}{6} + \frac{4\pi}{3} = \frac{9\pi}{6} \mathrel{\hat{=}} 270°$$

Lösungen:

$$x_1 = 3 \cdot e^{i30°}$$

$$x_2 = 3 \cdot e^{i150°}$$

$$x_3 = 3 \cdot e^{i270°}$$