

1) Bestimmen Sie die Lösung der folgenden LGS.

a)

$$\left( \begin{array}{ccc|c} 1 & 7 & 0 & 1 \\ 2 & 10 & 2 & 0 \\ 3 & 25 & 1 & 2 \\ 4 & 20 & 4 & 0 \end{array} \right) \begin{array}{l} IIa = II - 2 \cdot I \\ IIIa = III - 3 \cdot I \\ IVa = IV - 4 \cdot I \end{array}$$

$$\left( \begin{array}{ccc|c} 1 & 7 & 0 & 1 \\ 0 & -4 & 2 & -2 \\ 0 & 4 & 1 & -1 \\ 0 & -8 & 4 & -4 \end{array} \right) \begin{array}{l} IIIb = III + II \\ IVb = IV - 2 \cdot II \end{array}$$

$$\left( \begin{array}{ccc|c} 1 & 7 & 0 & 1 \\ 0 & -4 & 2 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \Rightarrow 3z = -3 \Rightarrow z = -1 \\ 0 = 0 \text{ gilt immer, Gleichung fällt weg} \end{array}$$

Aus II folgt:

$$\Rightarrow -4y + 2 \cdot (-1) = -2 \Rightarrow -4y = 0 \Rightarrow y = 0$$

Aus I folgt:

$$\Rightarrow x + 7 \cdot 0 = 1 \Rightarrow x = 1$$

Lösung:

$$x = 1$$

$$y = 0$$

$$z = -1$$

b)

$$\left( \begin{array}{ccc|c} 2 & 5 & 2 & -4 \\ -2 & 4 & -5 & -20 \\ 3 & -6 & 5 & 23 \end{array} \right) \begin{array}{l} IIa = II + I \\ IIIa = 2 \cdot III - 3 \cdot I \end{array}$$

$$\left( \begin{array}{ccc|c} 2 & 5 & 2 & -4 \\ 0 & 9 & -3 & -24 \\ 0 & -27 & 4 & 58 \end{array} \right) IIIb = IIIa + 3 \cdot IIa$$

$$\left( \begin{array}{ccc|c} 2 & 5 & 2 & -4 \\ 0 & 9 & -3 & -24 \\ 0 & 0 & -5 & -14 \end{array} \right) \Rightarrow z = \frac{14}{5}$$

Aus II folgt :

$$\Rightarrow 9y - 3 \cdot \left( \frac{14}{5} \right) = -24 \Rightarrow 9y - \frac{42}{5} = -\frac{120}{5} \Rightarrow y = -\frac{78}{5} \cdot \frac{1}{9} = -\frac{78}{45} = -\frac{26}{15}$$

Aus I folgt :

$$\Rightarrow 2x + 5 \cdot \left( -\frac{26}{15} \right) + 2 \cdot \left( \frac{14}{5} \right) = -4 \Rightarrow 2x = \frac{130}{15} - \frac{84}{15} - \frac{60}{15} = -\frac{14}{15} \Rightarrow x = -\frac{14}{15} \cdot \frac{1}{2} = -\frac{7}{15}$$

Lösung :

$$x = -\frac{7}{15}$$

$$y = -\frac{26}{15}$$

$$z = \frac{14}{5}$$

c)

$$\left( \begin{array}{ccc|c} 3 & 6 & -2 & -4 \\ 3 & 2 & 1 & 0 \\ \frac{3}{2} & 5 & -5 & -9 \end{array} \right) \begin{array}{l} IIa = II - I \\ IIIa = 2 \cdot III - I \end{array}$$

$$\left( \begin{array}{ccc|c} 3 & 6 & -2 & -4 \\ 0 & -4 & 3 & 4 \\ 0 & 4 & -8 & -14 \end{array} \right) IIIb = IIIa + II$$

$$\left( \begin{array}{ccc|c} 3 & 6 & -2 & -4 \\ 0 & -4 & 3 & 4 \\ 0 & 0 & -5 & -10 \end{array} \right) \Rightarrow z = 2$$

Aus II folgt :

$$\Rightarrow -4y + 3 \cdot 2 = 4 \Rightarrow -y = -2 \Rightarrow y = \frac{1}{2}$$

Aus I folgt :

$$\Rightarrow 3x + 6 \cdot \left(\frac{1}{2}\right) - 2 \cdot 2 = -4 \Rightarrow 3x = -3 \Rightarrow x = -1$$

Lösung :

$$x = -1$$

$$y = \frac{1}{2}$$

$$z = 2$$

d)

$$\left( \begin{array}{cccc|c} 3 & -1 & 4 & -2 & -8 \\ 2 & 1 & 1 & 0 & 1 \\ 0 & -3 & 4 & 2 & 5 \\ 7 & 1 & -1 & -2 & 15 \end{array} \right) \begin{array}{l} IIa = 3 \cdot II - 2 \cdot I \\ IVa = 3 \cdot IV - 7 \cdot I \end{array}$$

$$\left( \begin{array}{cccc|c} 3 & -1 & 4 & -2 & -8 \\ 0 & 5 & -5 & 4 & 19 \\ 0 & -3 & 4 & 2 & 5 \\ 0 & 10 & -31 & 8 & 101 \end{array} \right) \begin{array}{l} IIIa = 5 \cdot III + 3 \cdot IIa \\ IVb = IVa - 2 \cdot IIa \end{array}$$

$$\left( \begin{array}{cccc|c} 3 & -1 & 4 & -2 & -8 \\ 0 & 5 & -5 & 4 & 19 \\ 0 & 0 & 5 & 22 & 82 \\ 0 & 0 & -21 & 0 & 63 \end{array} \right) \Rightarrow y = -3$$

Aus III folgt :

$$\Rightarrow 5 \cdot (-3) + 22z = 82 \Rightarrow 22z = 97 \Rightarrow z = \frac{97}{22}$$

Aus II folgt :

$$\Rightarrow 5x - 5 \cdot (-3) + 4 \cdot \left( \frac{97}{22} \right) = 19 \Rightarrow 5x = \frac{418}{22} - \frac{388}{22} - \frac{330}{22} = -\frac{300}{22}$$

$$\Rightarrow x = -\frac{300}{22} \cdot \frac{1}{5} = -\frac{60}{22} = -\frac{30}{11}$$

Aus I folgt :

$$\Rightarrow 3w - \left( -\frac{30}{11} \right) + 4 \cdot (-3) - 2 \cdot \left( \frac{97}{22} \right) = -8 \Rightarrow 3w = -\frac{176}{22} - \frac{60}{22} + \frac{264}{22} + \frac{194}{22} = \frac{222}{22}$$

$$\Rightarrow w = \frac{222}{22} \cdot \frac{1}{3} = \frac{222}{66} = \frac{37}{11}$$

Lösung :

$$w = \frac{37}{11}$$

$$x = -\frac{30}{11}$$

$$y = -3$$

$$z = \frac{97}{22}$$

e)

$$\left( \begin{array}{cc|c} 2 & -7 & 9 \\ 11 & 5 & 6 \\ 3 & -7 & 10 \end{array} \right) \begin{array}{l} IIa = 2 \cdot II - 11 \cdot I \\ IIIa = 2 \cdot III - 3 \cdot I \end{array}$$

$$\left( \begin{array}{cc|c} 2 & -7 & 9 \\ 0 & 87 & -87 \\ 0 & 7 & -7 \end{array} \right) \begin{array}{l} \Rightarrow y = -1 \\ \Rightarrow y = -1 \end{array}$$

*Aus I folgt :*

$$\Rightarrow 2x - 7 \cdot (-1) = 9 \Rightarrow 2x = 2 \Rightarrow x = 1$$

*Lösung :*

$$x = 1$$

$$y = -1$$